Graph-based Pigment Network Detection in Skin Images

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Contents

Abstract

Since melanoma can only be treated in the first stages, it is very important to detect melanoma as early as possible. Such method as dermoscopy make it possible to scan for melanoma on a regular basis without biopsy. In particular, this technique allows examination of skin structures and patterns, and especially the identification of the most important feature of melanocytic lesions, known as pigment network. In this work I review a novel graph-based pigment network detection method that can find and visualize round structures belonging to the pigment network in dermoscopic images of skin lesions. The authors claim a very high efficiency of the method. In order to analyze the algorithm in detail I implemented the method following the steps described in the article. Arising problems led me to the conclusion, that the proposed algorithm in the paper is not suitable for clinical use as described by the authors.

Keywords: Dermoscopy, Skin lesion, Pigment network detection, Texture analysis, Graph, Cyclic subgraph

1 Introduction

Although malignant melanoma is less common than non-melanoma skin cancer, it is known to be one of the most lethal types of the skin cancers, causing more than 75 percent of all skin cancer deaths [1]. World Health Organisation estimates 132,000 new cases of melanoma per year internationally. In general, melanoma-related mortality rates continue to climb in line with increasing worldwide melanoma incidence [2]. Concerning melanoma mortality, Europe's rate of 1.5/100 000 is the third highest in the World, after Australia/New Zealand (3.5/100,000) and North America (1.7/100,000). Among the four geographical regions of Europe, the CEE countries have the largest share (35.5%) of the more than 20,000 melanoma deaths estimated to occur annually throughout the continent [3].

Melanomas can occur in different body organs (e.g., eyes or nails) but this work focuses on skin melanoma. Patients with this kind of melanoma usually present with skin lesions that have changed in size, color, contour, or configuration. Those changes serve as a first alarm for melanoma detection, and the acronym "'ABCDE"' is used to remember those physical characteristics suggestive of malignancy, which are, in other words, **A**symmetry, irregular **B**orders, **C**olor variations (especially red, white, and blue tones in a brown or black lesion), **D**iameter greater than 6 mm, and **E**volving over time [4]. Sometimes only first 4 characteristics are taken into consideration, and in that case the rule is called "'ABCD"' rule. It is very important to detect the "'ABCDE"' characteristics on early stage of melanoma.

The stage of a melanoma is an indicator of how deeply it has grown into the skin, and whether it has spread. Melanoma can be treated on early and medium stages by means of surgery: the skin cancer and some surrounding areas are removed [5]. The most difficult case (advanced stage) is metastasis, when melanoma spreads to other organs. When it happens, it usually cannot be cured and becomes fatal [6]. This explains the high necessity of melanoma recognition on early stages.

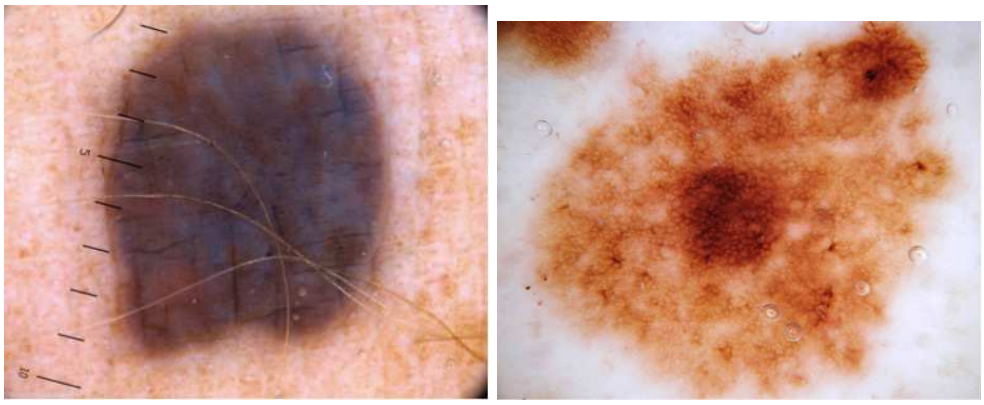
There are several ways to detect melanoma, for example, biopsy and imaging (radiology) tests. In the case of biopsy a sample of skin is taken and analyzed under a microscope. Experts look at certain features such as the tumor thickness and the portion of cells that are actively dividing. There are different ways to do a skin biopsy and all methods are likely to leave at least a small scar. Imaging (radiology) tests like chest x-ray, magnetic resonance imaging, positron emission tomography or computed tomography scans are done to create pictures of the inside of the body. They are used to look for the spread of melanoma and are not appropriate for people with very early melanoma, which is not likely to have spread. Blood tests are not used to find melanoma, but some tests may be done before or during treatment, especially for more advanced melanomas. Experts often test for blood levels of lactate dehydrogenase (LDH) before treatment. If the melanoma has spread to distant parts of the body, a higher than normal level of LDH is a sign that the cancer may be harder to treat [7]. Obviously, all those methods require work of experts and specific equipment.

In recent years a new approach for melanoma detection is being developed. It's based on detection of a pigment network in dermoscopic images of skin lesions. Dermoscopy (also known as dermatoscopy or epiluminescence microscopy) is a non-invasive method that allows the in vivo evaluation of colors and microstructures of the epidermis, the dermo-epidermal junction, and the papillary dermis not visible to the naked eye. This technique refers to the examination of the skin using skin surface microscopy. Dermoscopy requires a high quality magnifying lens and a powerful lighting system. This allows examination of skin structures and patterns. Several different lightweight, battery-powered hand-held devices are available at

the moment. Convenient attachments allow video or still photography. Computer software can be used to archive dermoscopy images and allow expert diagnosis. Computer aided diagnosis (CAD) programs may aid in diagnosis by comparing the new image with stored cases with typical features of benign and malignant pigmented skin lesions [8].

The most important feature of melanocytic lesions is the pigment network, which consists of pigmented network lines and hypopigmented holes. The network of hypopigmented holes corresponds to the suprapapillary plate, which is relatively thin and contains less melanin. The network lines correspond to the "'rete ridges"', which are thicker and have a greater quantity of melanin.

In normal melanocytic nevi, the pigment network is slightly pigmented. Light-brown network lines are thin and fade gradually at the periphery, holes are regular and narrow, the distribution is symmetric and sometimes accentuated in the center of the lesion. In cutaneous malignant melanoma, the pigment network usually ends abruptly at the periphery and has irregular holes, thickened and darkened network lines, and tree-like branching at the periphery. Moreover, the pigment network features change between different sectors of the pigmented skin lesion edge (border): some areas of malignant lesions manifest as a broad and prominent pigment network, while others have a discrete irregular pigment network; the pigment network may also be totally absent in some areas. Clinically atypical nevi (i.e., the nevi defined as dysplastic nevi at pathology evaluation) are often identified because they show areas of irregular and discrete pigment network, distributed asymmetrically (Figure 1.1) [9].



(a) (b)

Figure 1.1: (a) An image of a lesion without pigment network. (b) A lesion containing a pigment network.

There are exist different approaches for detection of pigment network in dermoscopic images. In this work I am going to observe and analyze the method proposed by M. Sadeghi, M. Razmara, M. Ester, T. K. Lee and M. S. Atkins [10]. In their approach they chain the following sequence of steps in order to find and visualize round structures belonging to the pigment network:

1. Manual image segmentation by an expert;
2. Highlighting the texture features;
3. Converting the image into a luminant;
4. Detecting sharp change of intensity;
5. Converting the resulting binary image to a graph;
6. Detecting all cyclic sub-graphs;
7. Removing noise or undesired cycles discriminating globules from meshes of the network;
8. Connecting the detected cyclic structures into a new graph;
9. Classifying the image as "'Absent"' or "'Present"' according to the density ratio of the new graph of the pigment network.

Being "'Present"' means that a pigment network is detected in the skin lesion and there is a high probability that the patient has melanoma.

The authors claim the accuracy of the method as 92.6% in classifying images to two classes of "'network is absent"' and "'network is present"'. Comparing to another approaches, it is a very high result.

2 Existing approaches observation

For this work, I have observed and compared several methods for melanoma detection by means of image processing, proposed by different authors. Some of the approaches are also based on pigment network detection, while others operate with different methods, like, for example, "'ABCD"' rule or decision trees. Almost all of the observed methods operate with epiluminescence microscopic (ELM) images, when one of the observed approaches work with images, produced with a consumer level camera. In 50% of approaches the lesion is segmented from the healthy skin manually, while others have derived special algorithms, giving good results in the segmentation. I also was surprised, realizing that in a pre-processing phase, only one of the observed systems consider removal of artifacts such as hair or reflected light. A brief description of each observed approach follows.

One of the oldest approaches, I have found, is the method proposed by S. Fischer et al. in 1996 [11]. In this method a color based segmentation applied to the Karhunen-Loeve transform of the RGB color vectors to separate brightness information or enhancement of structures with a poor contrast, then a scale-space filter is applied to every histogram to calculate fingerprints that allow for determination of color classes. A coarse classification assigns pixels in intervals to a class defined by a maximum histogram, unclassified pixels are then assigned to the closest class using the fuzzy c-means technique. After that they apply local histogram equalization to enhance the pigmented network homogeneously. And finally, to smooth out impulsive noise without destroying the structure of the pigmented network, morphological grey scale closing followed by an opening. Their experiments demonstrate that using a color based segmentation does not allow for the extraction of pigmented networks because of the weak contrast within the network. However, the authors claim that it is a robust method for separating lesions from the surrounding skin and for extracting homogeneous and differently colored regions within the lesion.

In 2004 T. Tanaka et al. published the paper "'A Study on the Image Diagnosis of Melanoma"' [12], dealing with features of melanoma and nevus for computer diagnosis. For melanoma detection they do not detect a network in images, but consider one hundred five values of features based on "'ABCD"' rule. However, this work is interesting from the point of contour extracting of lesioned region, where they get good results in segmentation of images.

G. Di Leo et al. in 2008 [13] succeeded in their work to detect whether pigmented network is Atypical, Typical or Absent applying decision-tree classification techniques to the results of specific image processing algorithms. As in many other approaches, their method is divided in the sequence of steps: I) image segmentation; II) pigment network detection; III) pigment network classification. For separation of the healthy skin from lesion they do sequentially color to monochrome image conversion obtaining three different monochrome images from the source image corresponding to the red, green and blue color components, respectively; image binarization using an adaptive threshold and, finally, border identification with an adopted simple blob-finding algorithm. Structures of pigmented network are detected mainly by searching for the occurrence of texture and successively by evaluating its possible chromatic and spatial distribution. 13 features have been extracted from each network. The solutions are represented by Decision Tree Classifiers. Given a large training set, in fact, decision tree classifiers could produce rules that perform well on the training data but do not generalize well to unseen data. Sensibility and specificity are both greater than 85%.

One year later Jose Fernandez Alcon et al. [14] have designed and implemented an automatic imaging system with decision support for inspection of pigmented skin lesions and melanoma diagnosis. They do not detect pigmented network either, but follow the "'ABCD"' rule. However the work is remarkable because it supports images of skin lesions acquired using a conventional (consumer level) digital camera. Authors in their method start with background correction by simply removing the low frequent spatial component of the image to correct for the uneven illumination. To separate the pigmented lesion from the background automatically they derived an algorithm inspired by Otsu's thresholding algorithm, which relies on maximizing the between-class variance. In order to decide whether the lesion is benign or malignant they follow the "'ABCD"' rule of dermatoscopy: scores and weights of "'ABCD"' features applying algorithms for the features quantification. The system classified images with an accuracy of 86%, a sensitivity of 94%, and a specificity of 68%.

Some works were published after the observed one. Catarina Barata et al. first published their work in 2011 [15] and updated their approach in 2012 [16]. They base their approach on a bank of directional filters. The latest system is able to detect, whether a skin lesion is with or without pigment network with sensitivity of 91.1% and and specificity of 82.1%. In a pre-processing phase the system detects artifacts such as hair or reflected light and removes them from the image using directional filters to reduce the influence of artifacts and avoid false alarms. Next, regions with pigment network are detected using two of its distinctive properties: intensity (i.e., the transitions between the dark lines and the lighter "'holes"') and geometry or spatial organization (since it is assumed that the lines of pigment network form connected structures). The intensity property is used to perform an enhancement of the network by applying a bank of directional filters, while the spatial organization is used to perform the actual detection and generate a binary "'net-mask"'. The final block aims to assign a binary label to each image: with or without pigment network. To accomplish this objective, features which characterize the topology of the detected regions in a given image are extracted and used to train a classifier, using a boosting algorithm. The authors claim that their system is different from the others, because it is able to: 1) report quantitative validation, 2) present qualitative results for the location of network, 3) extract the actual network (the mesh), and 4) compare the detected regions with a ground truth.

Finally, in August, 2012, there was an article of Leszek A. Nowak et al. published [17]. For detection process they have developed an adaptive filter, inspired by Swarm Intelligence (SI) optimization algorithms. The introduced filtering method is applied in a non-linear manner, to processed dermatoscopic image of a skin lesion. The non-linear approach derives from SI algorithms, and allows selective image filtering. In the beginning of filtration process, the filters (agents) are randomly applied to sections of the image, where each of them adapts its output based on the neighborhood surrounding it. Agents share their information with other agents that are located in immediate vicinity. This is a new approach to the problem of dermatoscopic structure detection, and the authors state it to be highly flexible, as it can be applied to images without the need of previous pre-processing stage. However, the method inherited high computation complexity of the optimization problems and this makes it very difficult to develop and fine tune, as processing one image can take up to 5 minutes depending on the agents count, iterations and image size.

3 Implementation of the observed method

To see how the system works practically I decided to implement the method following the steps, described in the article. For my experiments I have chosen Matlab (MathWorks, version R2012b) for the Image Processing Toolbox, providing the users with high range of functions for image analysis.

In the given approach the following phases can be distinguished: image pre-processing, network detection and, finally, classification. Pre-processing phase includes manual segmentation of the lesion, color enhancement, conversion to the luminance image, and detection of sharp changes of intensity. At the network detection phase the holes belonging to the pigment network pattern are connected together in a graph. And, after all, the resulting graph is analyzed and the presence of pigmented network in the image is accepted or rejected.

As the authors of the observed approach stated that they used random images from the Atlas of Dermoscopy [18], for my experiments I also have taken 5 images known to be melanoma positive from the same source (Figure 1.2). I have selected only pure images without noise and artifacts like hair or light reflections for accuracy of my experiment.

As it was said in the paper, the authors used images of size 768x512px, so I scaled the images to this size, using Adobe Photoshop.

I also used Adobe Photoshop for manual segmentation, filling the regions, belonging to the free of lesion skin with black color (Figure 1.3). As I am not an expert in dermatology, that was very challenging for me to classify, whether the structure shall be accepted or rejected, which explains the need of automatical detection.

For the contrast enhancement I used the Matlab function $h = fspecial(type,\ parameters)$ (Figure 1.4). As it was described in the paper: "'A 3-by-3 contrast enhancement filter created from the negative of the Laplacian filter with parameter α is used. α controls the shape of the Laplacian and must be in the range 0.0 to 1.0. Our default value for α is 0.2"'. That corresponds to $h = fspecial('unsharp',\ 0.2)$.

To convert the image to grayscale I used the Matlab function $G = rgb2gray(RGB)$, which converts RGB values to grayscale values by forming a weighted sum of the R, G, and B components: $0.2989 * R + 0.5870 * G + 0.1140 * B$. The same parameters were mentioned in the observed article (Figure 1.5).

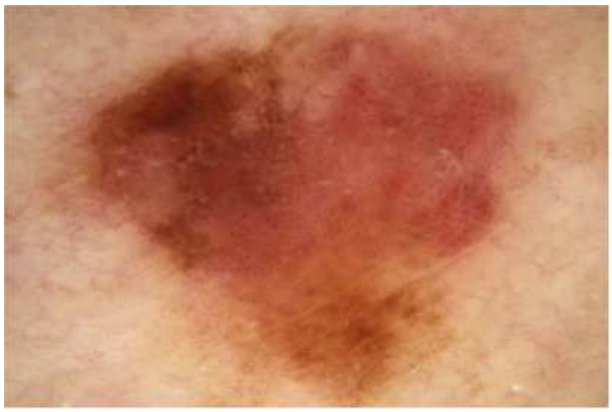


Figure 1.2: One of the images chosen for my experiments: original photo of a melanoma lesion.

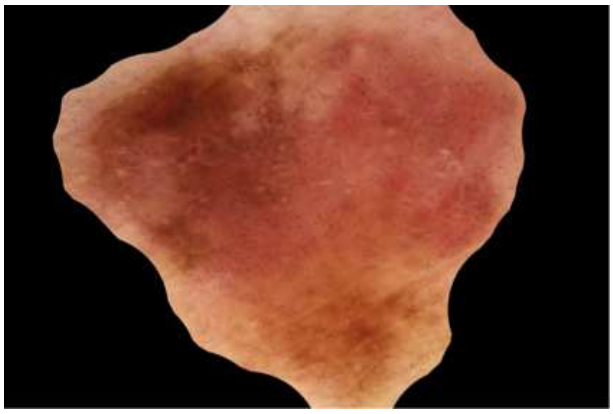


Figure 1.3: Segmented image of a melanoma lesion.

For the detection of sharp changes of intensity and transformation into a binary image the authors refer to the Laplacian of Gaussian (LoG) filter, which can detect the "'light-dark-light"' changes of the intensity well. The detection criterion is the presence of a zero crossing in the second derivative with the corresponding large peak in the first derivative. LoG looks for zero crossings and their transposes. All zeros are kept and edges lie on the zero points. If there is no zero, it arbitrarily chooses the edge to be the negative point. Therefore, when all zero responses of the filtered image are selected, the output image includes all closed contours of the zero crossing locations. The Matlab function *BW = EDGE(I,'log',THRESH,SIGMA)* was used for this purpose. The authors of the observed approach set up the threshold to zero. As the value for the sigma was not mentioned, I used the default value, which is equal to 2 (Figure 1.6).

Having a binary image of the connected components (the edges of the images), the authors convert them to a Graph (Gi) with 8-connected neighbors. Each pixel in the connected component is a node of Gi and each node has a unique label according to its coordinate.

In order to do that, the components of the binary image were labeled using the function *L = bwlabel(BW, n)*, returning a matrix L, of the same size as BW, containing labels for the connected objects in BW. The variable n can have a value of either 4 or 8, where 4 specifies 4-connected objects and 8 specifies 8-connected objects. If the argument is omitted, it defaults to 8. After that a function running through all labels and returning a list of edges for each connected component was written.

Having the list of edges as an input, it is possible to utilize the Iterative Loop Counting Algorithm [19], used by the authors of the approach, who state that morphologic techniques, used in the previous approaches are error-prone in detecting the round shape structures and therefore ILCA is a better tool to use. However, in my case it turned to be very slow for detection of structures in the whole image, and for that reason I reduced the whole number of labeles (connected components) to the certain range to continue experiment. I modified the ILCA that way, so it became possible to get the values of connected edges directly to the function, avoiding storing of intermediate values in a text file, however, it did not speed up the process very much. Found cyclic components are labeled with green (Figure 1.7).

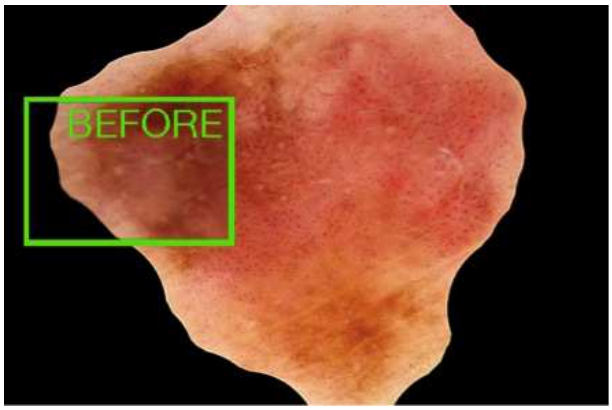


Figure 1.4: Image of a melanoma lesion with contrast enhancement implemented.

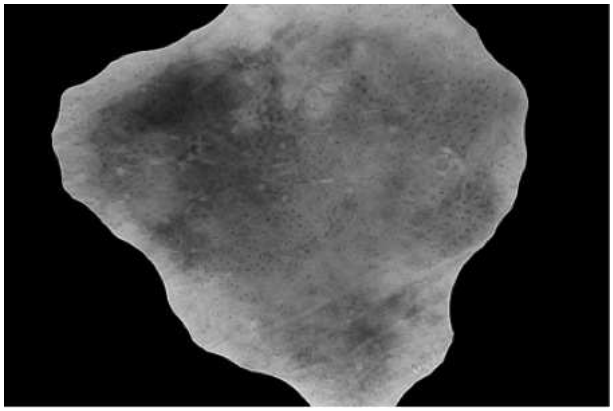


Figure 1.5: Image of a melanoma lesion converted to grayscale.

After receiving the list of edges contained in a loop, cyclic components are filtered according to the fact that in globules color of the inside area of the structure is darker than the border pixels or the outside area to avoid influence of noise such as hair or oil. To compare levels of intensity I calculate mean values of colors in grayscale for the loop area and the loop area extended by 2 pixels. If the loop area has larger intensity than extended area, the component is rejected. Rejected components marked with red color (Figure 1.8).

After identifying the holes, we are prepared to detect the presence of a pigment network pattern, creating a new graph. The cyclic structures become nodes of the new graph. Considering spatial arrangement of holes of the pigment network, a threshold for their Euclidean distance is set. Nodes within a maximum distance threshold (MDT) are connected together. The value of the MDT is computed based on the average diameter of all meshes in the image. The MDT should be proportional to the size of cyclic structures and it is defined as alpha (set to 3) times the average diameter of meshes. In order to calculate the average diameter of the meshes, it is used the parameter 'EquivDiameter' of the Matlab function "'regionprops"': *diameter = regionprops(BW,'EquivDiameter')*. It provides a scalar that specifies the diameter of a circle with the same area (*Area*) as the region with the Equation 1.1. The searched diameter is obtained by averaging the resulting vector.

$$diameter = \sqrt{\frac{4 * Area}{\pi}} \tag{1.1}$$

The nodes of the graph are the centroids of the meshes. To find the coordinates of centers the function *center = regionprops(BW,'Centroid')* is used. Having the list of center coordinates makes possible to go through it and to calculate distances between the centers of meshes. Those distances are estimated as Euclidean Distances. For each pair of center coordinates the Euclidean formula is computed with Equation 1.2. The centers separated from each other by a distance lower than 3 times MDT are connected. The labels for connectivity are stored in an adjacency matrix.

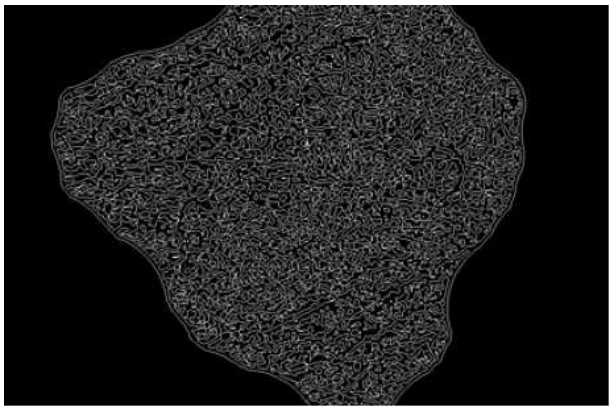


Figure 1.6: Image of a melanoma lesion transformed into a binary image.

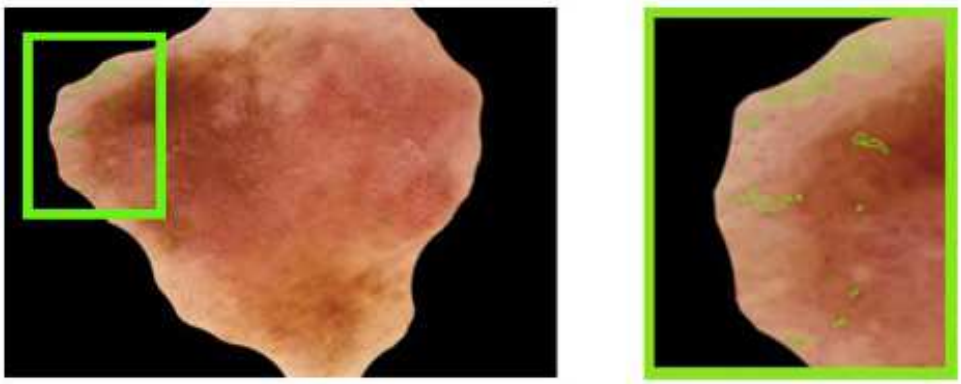


Figure 1.7: Cyclic structures found.

$$d(P1, P2) = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \tag{1.2}$$

Depending on the density of the graph, we can assume the presence of a pigment network. Density is calculated with the equation 1.3, where E is the number of edges in the graph, V is the number of nodes of the graph and *lesionSize* is the size of the area of the image within the lesion boundary being investigated.

$$density = \frac{|E|}{|V| * log(lesionSize)} \tag{1.3}$$

Images containing a density ratio higher than a threshold (set to 1.5) are classified as "'Present"' and in the opposite case - as "'Absent"'.

4 Results of my experiment

With Iterative Loop Counting Algorithm and hardware I used (Processor 1.7 GHz Intel Core i5, Memory 4Gb 1333 MHz DDR3) I failed to get any results for the whole image, because of the computation time. I tried to speed up the process, disregarding long loops in ILCA, setting up a treshold for number of edges to 100, what led me, obviously, to false results. The pigmented network in the processed images (Figure 1.9) was classified as Absent in cases known to be melanoma positive.

5 Discussion

Having my experiments done, I found some issues in the observed article.

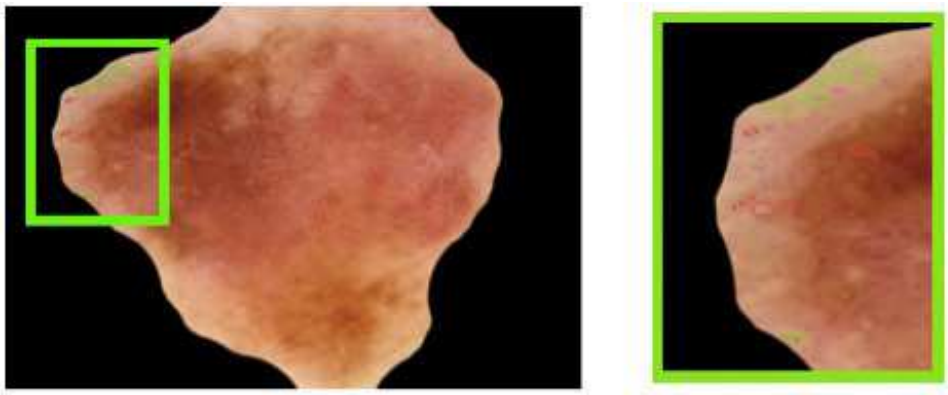


Figure 1.8: Filtering of cyclic structures.

One important issue is lesion segmentation. It turned out to be complicated to do it manually without any prior knowledge of dermatology and using just a graphic editor like Adobe Photoshop which gives a very rough output. Moreover, existing approaches already offer techniques for segmentation, which give rather good results. So, it can be the way of improvement for the observed approach.

The Iterative Loop Counting algorithm turned out to be very slow, making the whole process inefficient. It takes hours to analyze a single image. Having a big number of edges in undirected graph leads to exponential worst-case running time. It was proven that the problem of finding in a graph a cycle cover of smallest total length is NP-hard [20]. For that reason, I wonder, how the authors managed to handle this. The authors refer to morphological techniques as error-prone in detecting the round shape structures. However, implementation of morphologic techniques can speed up the process extremely, because the algorithms used for are not so complex. Thus, to find the cyclic structures, it is possible to create a binary mask layer, by filling in the structures. Matlab function *BW2 = imfill(BW,locations,conn)* can be used. Next, we can filter this masque, according to the prior knowledge of what a hole is. Knowing that a hole is small and at the same time big enough not to be a dot, we can define a threshold for its size. Than we need to discriminate the dots, globules and other dark artifacts like hair or ruler mark lines. The technique used in the observed approach, comparing levels of darkness of the structure and its rim, is appropriate for that reason. However, we also need to identify oil bubbles and white cysts, which the observed approach fail to do. Those structures, oppositely, have a very high levels of intensity and, therefore, can be filtered according to some value of threshold, which needs to be defined experimentally.

Furthermore, the authors do not define a suitable scaling for the investigated images. It is unknown, whether resolution and magnification is the same for all images of the atlas. Obviously, the influence of scaling is sufficient for the thresholds and all images shall be normalized in the pre-processing phase.

I have also noticed that detection of structures in light images and dark images gives different results, which explains the need of unification of the contrast level and illumination correction.

It also became obvious that the method works only for certain types of skin lesions. And in most of those cases the network pattern can be seen without implementing any algorithms.

6 Conclusion

Despite on the high efficiency claimed by the authors and all the issues mentioned above into account, the observed approach seems to be not suited for clinical use and requires further investigations and improvements.

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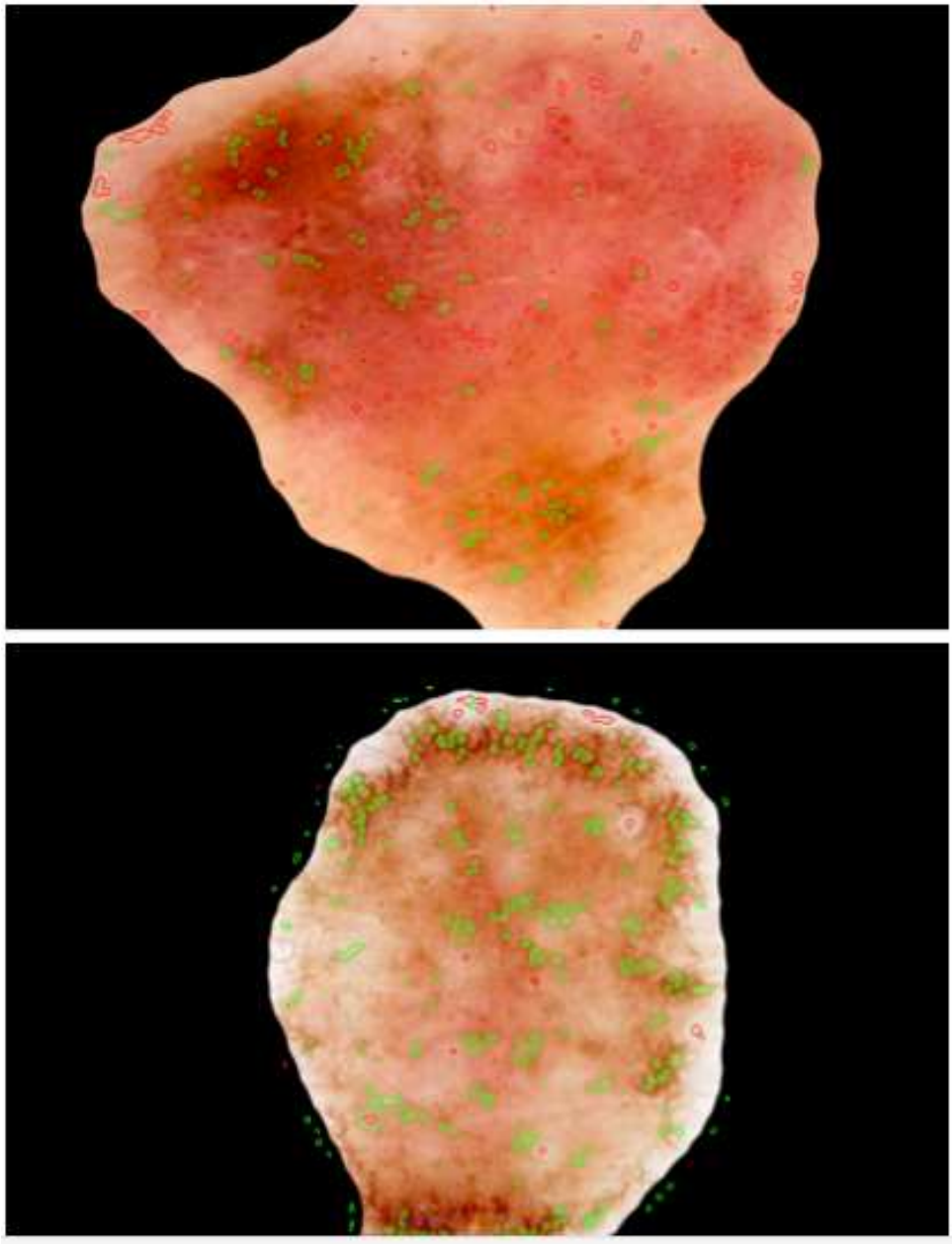

Figure 1.9: Output images of my experiment.